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DETERMINATION OF DYNAMIC YOUNG'S MODULUS FOR STEEL ALLOYS

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Abstract This paper presents an inverse problem for determining the dynamic Young's modulus for steel alloys and its variation with temperature. The theory is based on the Landau-Murnaghan constitutive law coupled with the longitudinal and flexural resonance methods. The results show that the Young's modulus varies in a complex way with temperature, in agreement with experimental results.

Key words: Young's modulus, Landau-Murnaghan constitutive law, resonance method, inverse problem.

1. INTRODUCTION

The variation with temperature of the Young's modulus E is not only a source of knowledge for the complex behaviour of materials, but also for understanding the structure-property relationship [1]. At high temperatures, the creep-failure tests become improper for evaluation the strength [2, 3] because the variation with temperature of the modulus E is intimately related to the internal structure of materials at atomic and microstructural levels.

In this paper, the nonlinear behaviour of the steel alloys is described by using the Lagrangian description where the state before deformation is used as a reference, and Landau-Murnaghan (LM) finite deformation theory for the unidirectional deformed isotropic solid [4-7] coupled with the resonance methods. By choosing the LM model it is possible to describe some features unknown in linear media such as the excitation of transverse component of the longitudinal wave propagating in structural steel alloys and subharmonic generation of waves.

Let us denote (u,v) the displacement vector, and (x,y,t) a point of observation in space and time. The equations of motion of a nonlinear elastic LM medium are given by

$$\rho\omega^2 u + F_1 + R_1 = 0, \tag{1}$$

$$\rho\omega^2 u + F_2 + R_2 = 0, \tag{2}$$

where ω is the circular frequency, ρ the density, and

$$F_1 = (\lambda+2\mu)u_{,xx} + \lambda v_{,xy} + \mu(u_{,yy} + v_{,xy}),$$

$$F_1 = \mu(u_{,xx} + v_{,xx}) + \lambda \mathrm{u}_{,xy} + \mu(u_{,xy} + v_{,xy}),$$

$$R_1 = c_1 u_{,x} u_{,xx},$$

$$R_2 = \mu(u_{,xx}u_{,y} + u_{,x}u_{,xy}) + (\lambda + 6C)u_{,x}u_{,xy} .$$

Here, λ and μ are Lamé's elastic constants related to Young's modulus by relation $E = \dfrac{\lambda(1+\nu)(1-2\nu)}{\nu}$, ν the Poisson's ratio, $c_1 = 3(\lambda+2\mu)+6C$ and C is the Landau elastic modulus of the 3-rd order [8] .

To relate (1) and (2) to classical 1D longitudinal wave equation [8], we write

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + M(u,v,x,t) , \tag{3}$$

where $c = \sqrt{E/\rho}$, or $c = f\bar{\lambda}$ if related to frequency and wavelength, and M a function which can be easily identified. We must say that the density is a function of temperature $\rho(T)$. By introducing a reference density ρ_0, we define a dimensionless density as $\tilde{\rho} = \dfrac{\rho}{\rho_0}$.

If the specimen has the length equals to $l = \bar{\lambda}/2$, we obtain for longitudinal vibrations

$$E = 4\rho l^2 f^2 + 4\rho l^4 f^3 C / 3h(l, f, \tilde{\rho})c , \tag{4}$$

where $h(l, f, \tilde{\rho})$ is a control function that depend on temperature.

In the case of flexural behaviour, the Bernoulli-Euler equation [9, 10] gives

$$E = \frac{64\pi^2 \rho l^4 f_n^2}{m_n^4 d^2} + \frac{8\pi^2 \rho l^8 f_n^4}{m_n^4 d^2 h^2(l, f, \tilde{\rho})} , \tag{5}$$

where f_n is the resonant frequency of the nth mode of vibration, d diameter of the specimen and m_n is the nth root of equation $\cos m \cosh m = 1$.

Lord Rayleigh [11] and Timoshenko [12-14] introduce in the motion equations the effects of rotatory inertia and of shear deformation. In this case, the Young's modulus E is evaluated as

$$E = K_n f_n^2 T_n \frac{\rho l^4}{d^2} + 14 K_n f_n^5 T_n \frac{\rho l^{10}}{3d^2 h^3(l, f, \tilde{\rho})} , \tag{6}$$

with K_n is a constant which depend on the number of vibration mode, and T_n a constant which depend on the vibration mode, d/l, and the shear modulus G [2].

2. INVERSE PROBLEM

We assume that the function $h(l, f, \tilde{\rho})$ can be expressed as power series in terms of a small parameter $\delta = \dfrac{T}{mT_0}$, where T_0 is a reference temperature and m a given parameter

$$h(l, f, \tilde{\rho}) = \sum_{k=0}^{N} h^{(k)}(l, f, \tilde{\rho}) \delta^k . \tag{7}$$

$$h^{(1)}(l, f, \tilde{\rho}) = a_1(1 + \exp\theta_1) , \quad h^{(2)}(l, f, \tilde{\rho}) = a_2\left(1 + \exp\theta_1 + \exp\theta_2 + \exp(\theta_1 + \theta_2)\right) , \tag{8}$$

$$\ldots\ldots\ldots$$

$$h^{(N)}(l, f, \tilde{\rho}) = a_N \left(1 + \sum_{j=1}^{N} \exp\theta_j + \sum_{j \neq l = 1}^{N} \exp(\theta_j + \theta_l) + \sum_{j \neq l \neq r = 1}^{N} \exp(\theta_j + \theta_l + \theta_r) + ... \right),$$

$$\theta_k = b_k x + c_k y - \omega_k \tau + \varsigma_k \ , \quad k = 1, 2, ...N \,,$$

where b_k, c_k are dimensionless wave numbers, ω_k the dimensionless frequencies and ς_k the dimensionless phases. The series (7) and (8) inserted into (1) - (6) gives a set of equations obtained by equating the powers of δ . The parameters a_k, b_k, c_k , ω_k and ς_k , $k = 1, 2, ...N$, are computable from this set of equations and an inverse problem coupled with a genetic algorithm.

To extract the Young's modulus for a given material from (1)-(8), the least-squares optimisation technique is used. An objective function $\Im$ is chosen to measure the agreement between theoretical and experimental data

$$\Im(C) = \sum_{i=1}^{M} \{ [v_i^e - v_i \,(C)]^2 + \gamma^2 \}, \tag{9}$$

where v_i^e are the measured *i*'th Young's modulus, v_i the corresponding model prediction, M the number of measurements $M > p$, with p *is* the number of unknown parameters.

In (9) the quantity γ estimates the verification of the set of equations obtained by equating the powers of δ . The parameter vector C contains $5N$ unknowns, i.e. a_k, b_k, c_k , ω_k and ς_k , $k = 1, 2, ...N$. We define fitness as follows

$$F = \frac{\Im_0}{\Im} \,, \quad \Im_0 = \sum_{i=1}^{M} (v_i^e)^2 \,. \tag{10}$$

As the convergence criterion of iterative computations, the expression Z to be maximum is defined

$$Z = \frac{1}{2} \log_{10} \frac{\Im}{\Im_0} \ \to \ \max \,. \tag{11}$$

A binary vector with $5N$ genes is used to represent the real values of unknowns. The length of the vector depends on the required precision, which in this case is six places after the decimal point. The domain of parameters $C \in [-a_i, a_i]$, $i = 1, 2, ..., 5N$ is divided into a least 15000 equal size ranges. If $a_i = 100$ the length of this domain is 200. That means that each unknown is represented by a gene (string) of 22 bits ($2^{21} < 3000000 \leq 2^{22}$). One individual is consisted with the row of $5N$ genes, that is, a binary vector with components

$$(\, b_{21}^{(1)} b_{20}^{(1)} ... b_0^{(1)} b_{21}^{(2)} b_{20}^{(2)} ... b_0^{(2)} ... b_{21}^{(5N)} b_{20}^{(5N)} ... b_0^{(5N)} \,)$$

The mapping from this binary string into $5N$ real numbers from the range $[-100, 100]$ is completed in two steps [15-18]:

1. convert each string ($b_{21}^{(i)} b_{20}^{(i)} ... b_0^{(i)}$) from the base 2 to base 10

$$(b_{21}^{(i)} b_{20}^{(i)} . b_0^{(i)})_2 = C'_i \ , \ \ i = 1, 2, ..., 5N$$

2. find a corresponding real number C_i , $i = 1, 2, ..., 5N$.

3. RESULTS

Excitation of transverse component v of the longitudinal wave propagating in the steel alloys is given by (2). The wave field does not propagate only in the direction x but also in the direction y. As the result, the waves with doubled frequency are put into evidence. The relationship between the amplitudes u_0 and v_0 of this wave is given by

$$v_0 / u_0^2 = -\mathrm{i}c_1 h(l, f, \tilde{\rho}) / 3lf\rho a^3, \tag{12}$$

where $a = \sqrt{\dfrac{\lambda + 2\mu}{\rho}}$.

Consider the 4330V steel alloy. The specimen dimensions are $l =$ 76.36 mm, $d = 3$ mm and $\rho = 7840$ kg/m^3, at $T = 296\,K$ [2].

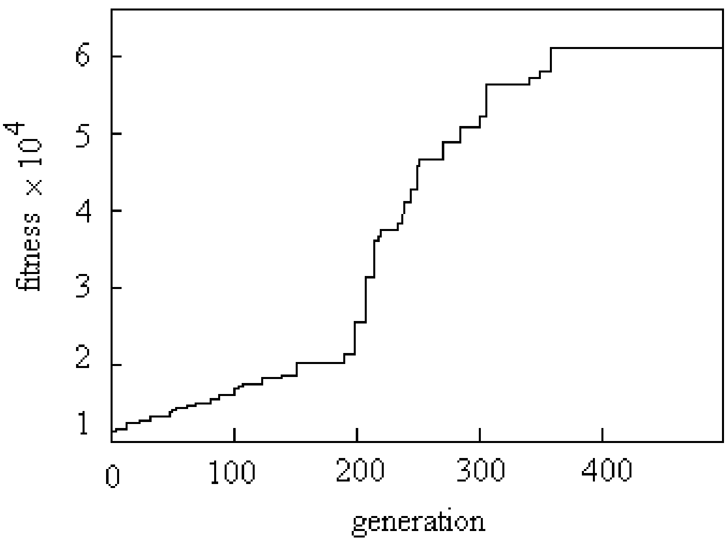

Fig. 1. Variation of maximum fitness for $\varepsilon = 0.01$.

Determination of the Young's modulus and its variation with the temperature is performed based on the above-mentioned results.

Comparing only the performance, the genetic algorithm is superior to other conjugate gradient methods because it is simple to be applied, is stable and the correct solutions are detected through a relatively small number of iterations, without requiring the stopping criterion for them. In order to analyse the effect of noise in the experimental data used in the inverse problem, the measured i'th Young's modulus v_i^e are multiplied by $(1 + r_i)$, where r_i are random numbers uniformly distributed in a given interval $[-\varepsilon, \varepsilon]$, with $\varepsilon = 0,\ 0.01,\ 0.1$. The final values of maximum fitness after 344 iterations are 3.45×10^4 for $\varepsilon = 0$, 7.02×10^3 for $\varepsilon = 0.01$ and respectively 0.88×10^2 for $\varepsilon = 0.1$. Thus, highly accurate measurements for Young's modulus are required to obtain good predictions. Even small perturbations in the Young's modulus values can lead to erroneous estimates.

Variation of the maximum of fitness for alternation of generations is given in Fig. 1, for $\varepsilon = 0.01$. The function h / h_0 is represented in Fig. 2, for $\tilde{\rho} = 1$, and h_0 a reference function for $\varepsilon = 0.01$. The variation of h / h_0 with respect to $\tilde{\rho}$ is displayed in Fig. 3 for some frequencies and their subharmnics, for $\varepsilon = 0.01$.

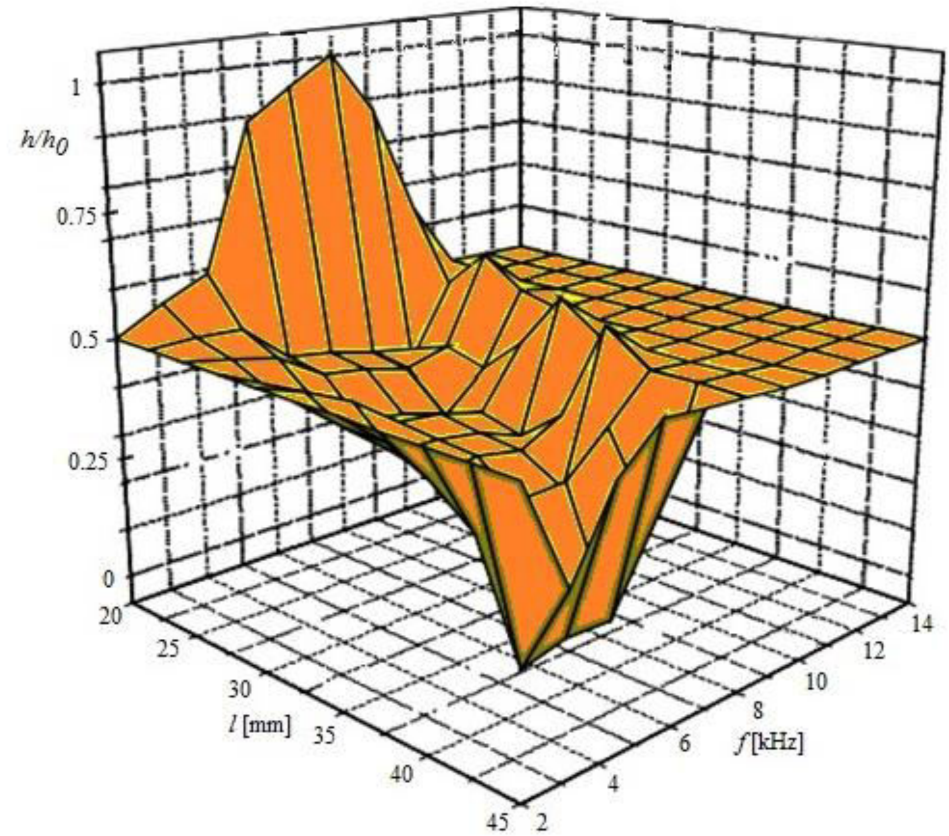

Fig. 2. Function h/h_0 for $\tilde{\rho}=1$.

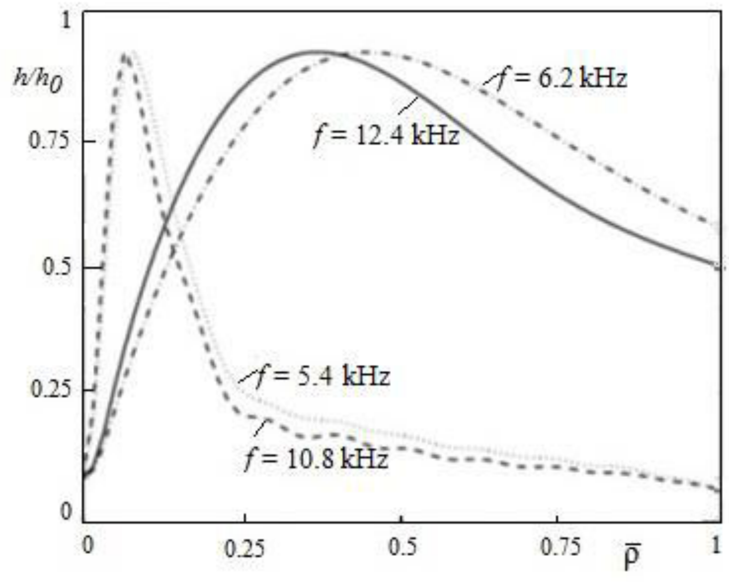

Fig. 3. Variation of h/h_0 with respect to $\tilde{\rho}$ for two frequencies and their subharmonics.

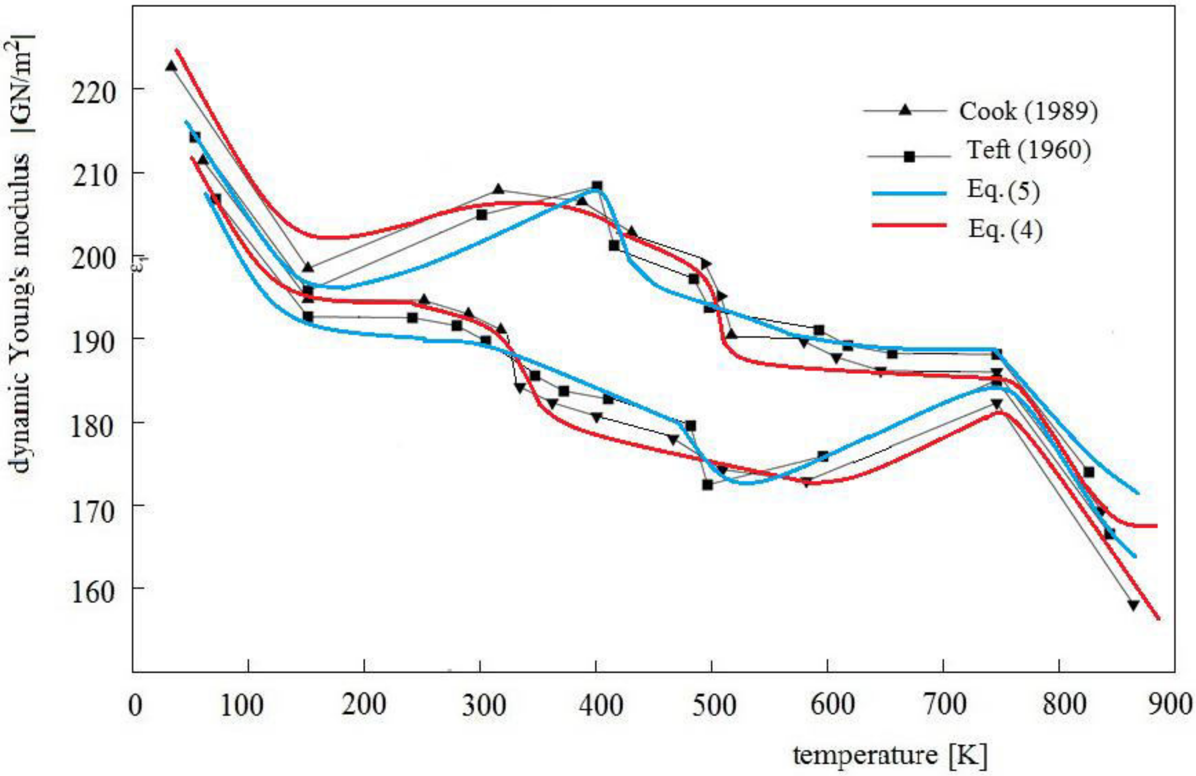

Fig. 4. Variation with temperature of the dynamic Young's modulus in 4330V steel, for longitudinal vibrations and flexural vibrations.

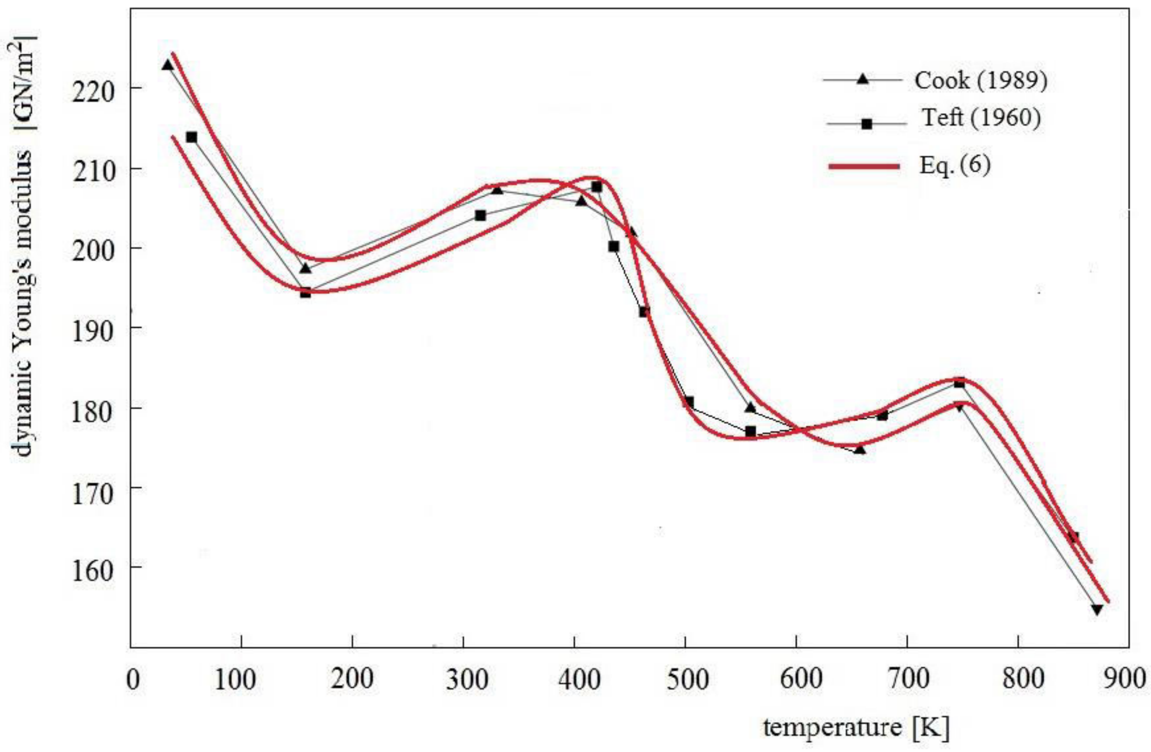

Fig. 5. Variation with temperature of the dynamic Young's modulus in 4330V steel, for flexural vibration in Timoshenko beam model.

Variation with temperature of the dynamic Young's modulus in 4330V steel is displayed in Fig. 4 for longitudinal vibrations and flexural vibrations in Bernoulli-Euler model.

Results are obtained by using equations (4) and (5), respectively, and compared with the experimental results given by Cook *et al.* [2] and Teft [14].

We see that both equations give results closest to the experimental results.

In the case of Timoshenko beam model, variation with temperature of the dynamic Young's modulus in 4330V steel is displayed in Fig. 5. The results are obtained by using equation (6) and compared with the experimental results given by Cook *et al.* [2] and Teft [14]. As above, we see that both equations give results closest to the experimental results.

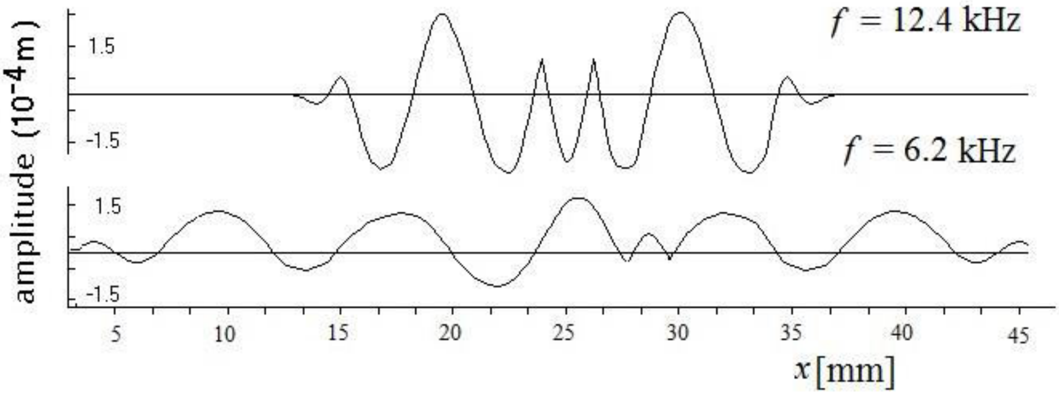

Fig. 6. The amplitudes of the displacement for $f = 6.2\text{kHz}$ and $f = 12.4\text{kHz}$.

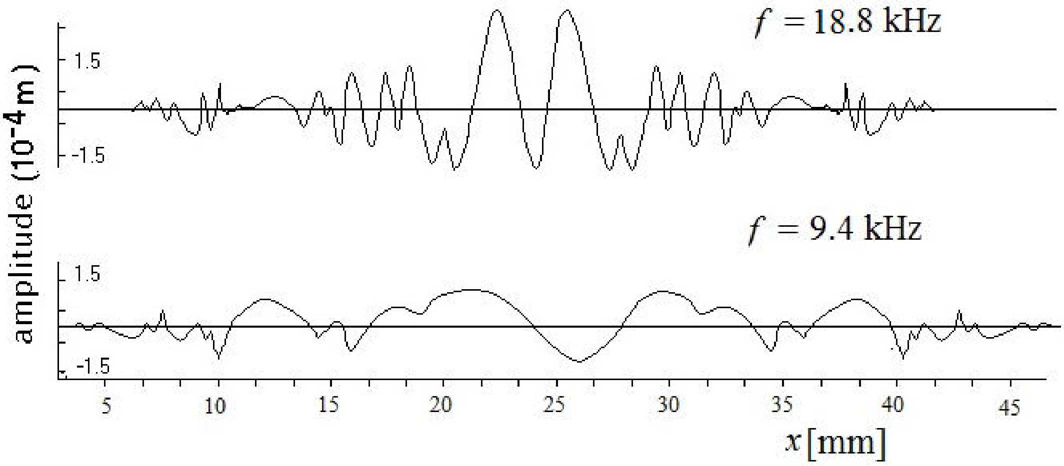

Fig. 7. The amplitudes of the displacement for $f = 9.4\text{kHz}$ and $f = 18.8\text{kHz}$.

The LM model describes the generation of subharmonics. As can be seen in Fig. 6 (and Fig.7, respectively), the displacements of the normal mode $f = 12.4\text{kHz}$ (18.8kHz) and respectively, of the subharmonic mode $f = 6.2\text{kHz}$ ($f = 9.4\text{kHz}$), for Timoshenko beam model, represent two kind of vibration regimes: a localised-mode (fracton) regime for $f = 12.4\text{kHz}$ (18.8kHz) and an extended-vibration (phonon) regime for $f = 6.2\text{kHz}$ ($f = 9.4\text{kHz}$). The fracton vibrations are mostly localised on small distances, while the phonon vibrations essentially extend to the whole beam.

In the case of a periodical plate the dispersion prevents good frequency matching between the fundamental and appropriate subharmonic modes [19]. Table 1 shows the computed frequencies for Timoshenko beam model. We see that for given frequency f_n, the generating of $f/2$ subharmonic is determined by the existence of a small frequency mismatch $f_n - f/2$ and large spatial overlap between the fundamental and subharmonic displacement field. By choosing the LM model, the excitation of transverse component of the longitudinal wave propagating in structural steel alloys is put into evidence. Fig. 8 shows the propagation in time of both components of displacement in Timoshenko beam model.

Table 1. Computed frequencies for Timoshenko beam model

f [kHz]	5.4 $\pm$ 0.05	6.2 $\pm$ 0.01	7.9 $\pm$ 0.03	8.4 $\pm$ 0.1	9.4 $\pm$ 0.01	10.8 $\pm$ 0.01	11.3 $\pm$ 0.1	12.4 $\pm$ 0.02	13.6 $\pm$ 0.03
	14.9 $\pm$ 0.01	15.8 $\pm$ 0.03	16,2 $\pm$ 0.06	16.8 $\pm$ 0.06	17.2 $\pm$ 0.1	17.5 $\pm$ 0.07	18.8 $\pm$ 0.02	19.9 $\pm$ 0.05	21.6 $\pm$ 0.4

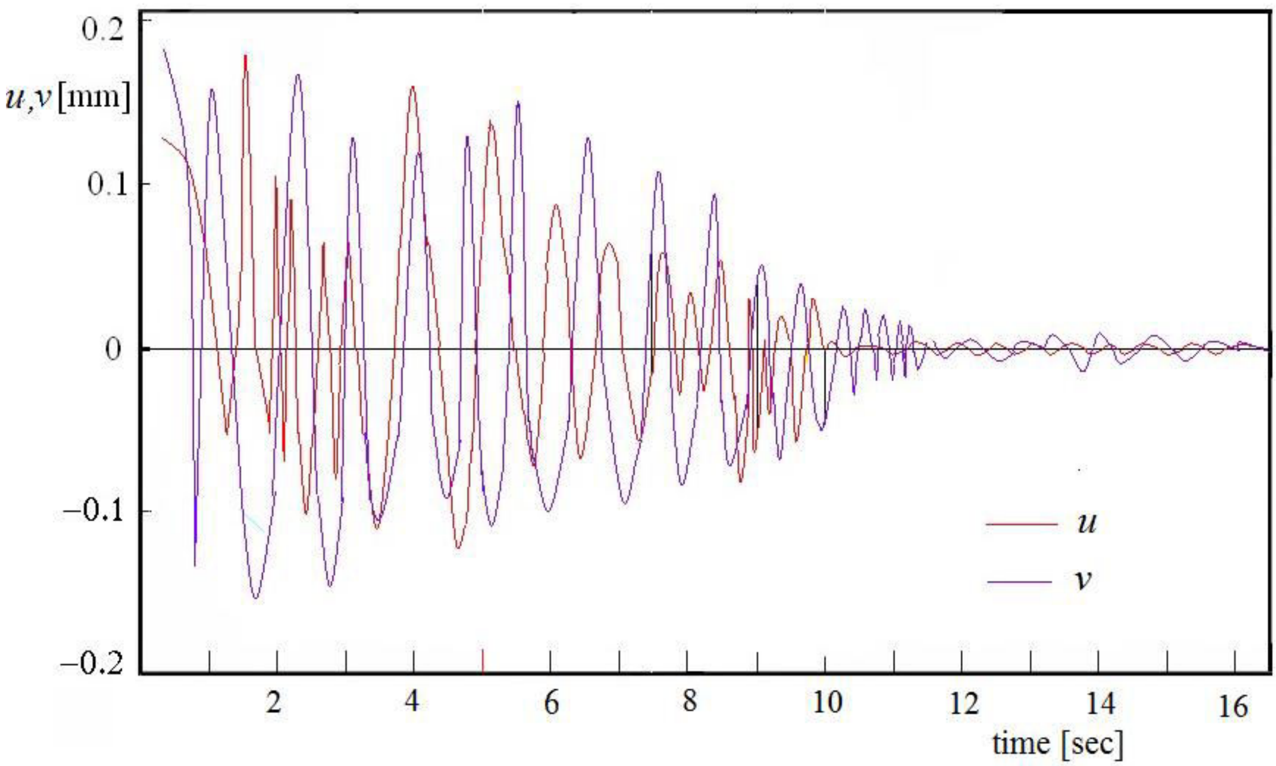

Fig. 8. Excitation of transverse component of the longitudinal wave propagating in Timoshenko beam model.

4. CONCLUSIONS

The dynamic Young's modulus for steel alloys and its variation with temperature is presented in this paper, based on the Landau-Murnaghan constitutive law coupled with the resonance methods and a genetic algorithm. The LM model is able to describe interesting phenomena unknown in linear media, such as the waves with doubled frequency with respect to the basic mode, or excitation of transverse component of the longitudinal waves in structural steel alloys. Variation with temperature of the dynamic Young's modulus in 4330V steel is analyzed for longitudinal vibrations, flexural vibrations in Bernoulli-Euler model, and flexural vibration in Timoshenko beam model, respectively. The application of our formulas in estimating the Young's modulus were tested by solving an inverse problem, for which an objective function was chosen to measure the agreement between theoretical and experimental data. The points to be elaborated are based on followings:

(a) use of the Lagrangian description for an unloaded non-deformed isotropic object;

(b) propagation of an elastic wave in a finitely deformed object loaded and stressed in the uniaxial direction.

(c) estimating the Young's modulus for the longitudinal vibrations, flexural vibrations in Bernoulli-Euler model, and flexural vibration in Timoshenko beam model, respectively, from the viewpoint of Murnaghans's finite deformation theory.

The LM model describes some features unknown in linear media such as the excitation of transverse component of the longitudinal wave propagating in structural steel alloys and subharmonic generation of waves.

The results show that the Young's modulus varies in a complex way with temperature, in agreement with experimental results.

REFERENCES

1. CHIROIU, C., MUNTEANU, L., CHIROIU, V., DELSANTO, P.P., SCALERANDI, M., *A genetic algorithm for determination of the elastic constants of a monoclinic crystal*, Inverse Problems, vol.16, pp.121-132, 2000.
2. COOK, L.S., WOLFENDEN, A., LUDTKA, G.M., *Longitudinal and flexural resonance methods for the determination of the variation with temperature of dynamic Young's modulus in 4330V steel*, Dynamic Elastic Modulus: Measurements in Materials, pp75-89, 1989.
3. WOLFENDEN, A., HARMOUCHE, M.R., BLESSING, G.V., CHEN, Y.T., TERRANOVA, P. et.al., *Dynamic young's modulus measurements in metallic materials*: *Results of an interlaboratory testing program*, Journal of Testing and Evaluation, vol.17, no.1, pp. 2-13, 1989.
4. LANDAU, L.D., RUMER, G., *UBERSCHALL ABSORPTION IN FESTERN KORPERN*, ZS. Sov. Phys., B3, 18–27, 1937.
5. MURNAGHAN, E.D., *Finite deformation of an elastic solid,* John Wiley, New York, 1951.
6. WHITHAM, G.B., *Linear and nonlinear waves,* John Wiley, New York, 1974.
7. MOLOTKOV, I. A., PSENCIL, I., *Longitudinal waves of a finite amplitude in nonlinear elastic media,* Seismic Waves in Complex 3-D Structures, Report 14 (Department of Geophysics, Charles University, Prague 2004), pp.197-202, 2004.
7. LANDAU,L.D., LIFSHITZ, E.M., *Theory of elasticity,* Pergamon Press, New York, 1969.
8. TIMOSHENKO, S., *Vibration problems in engineering*, Van Nostrand, New York, pp.199-326, 1928.
9. GOENS, V.E., *Uber die Bestimmung des Elastizitatsmodulus von Staben mit Hilfe von Biegungsschwingungen,* Annalen der Physik, 5. Folge, Band 11, Heft 6, 1931.
10. RAYLEIGH, *The theory of sound*, Macmillan, London, pp.359-360, 1926.
11. TIMOSHENKO, S., *On the correction for shear of the differential equation for transverse vibrations of prismatic bars*, Philosophical Magazine, vol. XLI-6th series, pp.744-747, 1921.
12. TIMOSHENKO, S., *On the transverse vibrations of bars of uniform cross-section*, Philosophical Magazine, vol. XLIII-6th series, pp.125-131, 1922.
13. MINDLIN, R,D., DERESIEWICZ, H., *Timoshenko's shear coefficient for flexural vibrations of beams*, Proceedings Second National Congress of Applied Mechanics, ASME, 1954.
14. TEFT, W.E., *Numerical solution of the frequency equations for the flexural standards*, vol.64B, no.4, pp. 237-242, 1960.

15. CHIROIU, V, DELSANTO, P. P., MUNTEANU, L, RUGINA, C., SCALERANDI, M., *Determination from the natural frequencies of the second and third order elastic constants in Al*, Journal of Acoustical Society of America, vol.102, pp.193–198, 1997.
16. DELSANTO, P. P., MOLDOVEANU, F., SCALERANDI, M., *A genetic algorithm technique for acoustic tomography*, Proceedings of the 8th Joint EPS-APS Int. Conf. on Physics Computing, Krakow, 1996.
17. CHIROIU, V., RUFFINO, E., SCALERANDI, M., MUNTEANU, L., *Determination of dynamic elasticity moduli in thin composite plates*, Revue Roumaine des Sciences Techniques – Série de Mécanique Appliquée, vol. 44, no. 1, 1999.
18. CHIROIU, V., MOLDOVEANU, F., CHIROIU, C., SCALERANDI, M., RUFFINO, E., *Application of genetic algorithm in defects visualization*, Revue Roumaine des Sciences Techniques – Série de Mécanique Appliquée, vol. 44, no. 2, 1999.
19. CHIROIU, C., DELSANTO, P.P., SCALERANDI, M., CHIROIU, V., SIRETEANU, T., S*ubharmonic generation in piezoelectrics with cantor-like structure,* Journal of Physics D: Applied Physics, Institute of Physics Publishing, 34, 3, 1579-1586, 2001.

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