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ON DECAY OF DILATATIONAL ACCELERATION WAVES IN POROUS THERMOELASTIC MATERIALS

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Abstract This paper is dedicated to a porous thermoelastic material, namely a material with voids having a micropolar structure. In the set of constitutive variables we included a new variable, namely voidage time derivative. For the mixed initial-boundary value problem within this context, we formulate and prove an uniqueness result regarding the solution. We also study the decay of acceleration waves in this bodies with pores.

Key words: Micropolar, Voids, Thermoelastic, Acceleration waves.

1. INTRODUCTION

The considerations rom our study can be useful in other fields of applications which deal with porous bodies, as solid packed granular, geological materials, and so on. The first investigations on bodies with voids were made by Goodman and Cowin in [1]. In this granular theory the authors introduce a new degree of freedom to develop the mechanical evolution of porous bodies in which the interstices are voids material and the matrix material is elastic. This procedure is included also in the paper [2] of Cowin and Nunziato. There are many applications of this new theory in the study of geological materials like soils and rocks and artificial manufactured porous materials, like pressed powders and ceramics. Specific for bodies with voids is the fact that density can be written as the product of two sizes namely, the volume fraction and the matrix material (see also, [4], [5], [6]). In the theory from works [1] and [2], as well as in [3], the thermal effect is not taken into account. That is why it appears natural the generalization proposed in [7] in which thermoelastic bodies are considered. But in this last paper it is neglected the fact that the variations in the volume fraction lead to an internal dissipation of energy in the body. Other results for bodies with microstructure can be found in [8-15]. Our intentions in this paper is to generalize the theory set forth in papers [1] - [3] to cover the theory of bodies with voids and micropolar structure. To this aim we consider a new independent variable, namely the time derivative of the voidage, which allows us to take into account the inelastic effects.

2. BASIC EQUATIONS

We consider a domain D from the Euclidean three-dimensional space R^3 which is occupied, at the initial moment $t = 0$, by a micropolar porous material. Assume that the border ∂D of D is a regular surface that allows the application of theorem of divergence.

Also, we denote the closure of B by $\overline{B}$. The motion of the bodies is referred to a system of Cartesian axes Ox_m; (i = 1, 2, 3) and the Cartesian vector and tensor notation is adopted. For the derivative with respect to time variable we will use a superposed dot and for the partial derivatives with respect to the spatial variables we will use a comma followed by a subscript: *f, i.* As usual, the Einstein summation rule is used, regarding the repeated indices.

In the initial state of our configuration, we have the following relation:

$$\rho_o = \gamma_o \nu_o, \tag{1}$$

where ρ is the bulk density, γ is the matrix density and ν is the volume fraction and suppose that ρ_o, γ_o and ν_o_0 are spatially constants.

In order to characterize the evolution of the thermoelastic micropolar body with voids, we will use the following independent variables:

- $v_m(x,t)$ - the components of the displacement vector;
- $\varphi_m(x,t)$ - the components of the microrotation vector;
- ϑ - the variation of the temperature from the initial temperature T_o;
- σ - the fraction of change in volume.

We will assume that our initial body has no stress, has no intrinsic equilibrated force and have no ux rate. For this reason and taking into account the fact that we will make the considerations only in the context of a linear theory, we deduce that it is very natural that the energy function can be written in the following form:

$$\begin{aligned}\rho\Psi &= \frac{1}{2}A_{ijmn}e_{ij}e_{mn} + B_{ijmn}e_{ij}\varepsilon_{mn} + \frac{1}{2}C_{ijmn}\varepsilon_{ij}\varepsilon_{mn} + B_{ij}\sigma e_{ijn} + C_{ij}\sigma\varepsilon_{ijn} + D_{ijk}\sigma_{,k}e_{ij} + E_{ijk}\sigma_{,k}\varepsilon_{ij} - \\ &- \beta_{ij}\vartheta e_{ij} - \alpha_{ij}\vartheta\varepsilon_{ij} - m\vartheta\sigma + d_i\sigma\sigma_{,i} + a\vartheta\sigma_{,i} - \frac{1}{2}a\vartheta^2 + \frac{1}{2}\xi\sigma^2 + \frac{1}{2}A_{ij}\sigma_{,i}\sigma_{,j} - \frac{1}{2}\omega\dot{\sigma}^2\end{aligned} \tag{2}$$

By using a suggestion given in [2], we will use the notation $f = -\omega\dot{\sigma}$ to designate the dissipation, as we already told, consider of the inelastic evolution of the pores. In this notation ω is considered a nonnegative constant. By using the energy function, with the help of the procedure from [1-3] we deduce the next constitutive relations, which give

$$\tau_{ij} = A_{ijmn}e_{mn} + B_{ijmn}\varepsilon_{mn} + B_{ij}\sigma + C_{ij}\sigma\varepsilon_{ijn} + D_{ijk}\sigma_{,k} - \beta_{ij}\vartheta, \tag{3}$$

$$\mu_{ij} = B_{mnij}e_{mn} + C_{ijmn}\varepsilon_{mn} + C_{ij}\sigma + E_{ijk}\sigma_{,k} - \alpha_{ij}\vartheta, \tag{4}$$

$$h_m = A_{ij}\sigma_{,j} + D_{mni}e_{mn} + E_{mni}\varepsilon_{mn} + d_m\sigma - a_m\vartheta, \tag{5}$$

$$g = -B_{ij}e_{ij} - C_{ij}\varepsilon_{ij} - \xi\sigma - d_m\sigma_{,i} + m\vartheta, \tag{6}$$

$$\eta = \beta_{ij}e_{ij} + \alpha_{ij}\varepsilon_{ij} + m\sigma + a_m\sigma_{,i} + c\vartheta, \tag{7}$$

$$q_m = k_{mn}\vartheta_{,n}, \tag{8}$$

where e_{mn} and ε_{mn} are the tensors of the strain and these are defined by means of the following kinematic equations:

$$e_{ij} = u_{j,i} + e_{jik}\varphi_k \quad , \quad \varepsilon_{ij} = \varphi_{j,i} \quad , \quad \vartheta = T - T_o \quad , \quad \sigma = \nu - \nu_o \ . \tag{9}$$

By using the procedure suggested in the paper [3] by Nunziato and Cowin, we can deduce the following basic equations:
- the motion equations:

$$\tau_{ij,j} + \rho F_m = \rho \ddot{u}_m \quad ; \tag{10}$$

$$\mu_{ij,j} + \varepsilon_{ijk}\tau_{jk} + \rho M_m = I_{ij}\ddot{\varphi}_j \, ; \tag{11}$$

- the equation of the equilibrated forces:

$$h_{i,i} + g + \rho L = \rho\kappa\ddot{\sigma} \, ; \tag{12}$$

- the equation of energy:

$$\rho T_o \dot{\eta} = q_{i,i} + \rho r \, . \tag{13}$$

The significance of the above notations is as follows:
ρ - density (which is constant);
η - entropy;
T_o - initial temperature (a positive constant);
I_{mn} - tensor of inertia;
κ - inertia of the equilibrated forces;
v_m - vector of displacement ;
φ_m - vector of microrotation;
φ - variation of the distribution initial state φ_o ;
σ - variation of the volume fraction from the initial state;
ϑ - variation of the temperature from the initial temperature T_o ;
e_{ij}, ε_{ij} - tensors of the strain;
τ_{ij} - stress tensor;
μ_{ij} - couple stress tensor;
h_m - vector of equlibrated stress;
q_m - vector of heat flux;
F_m - body forces;
M_m - body couple;
r - supply of heat;
g-the intrinsic equilibrated force;
L-the extrinsic force;
$A_{ijmn}, B_{ijmn}, ..., k_{ij}$ -the coefficients that characterize the elastic properties of the material which the following relations of symmetry:

$$A_{ijmn} = A_{mnij} \, , C_{ijmn} = C_{mnij} \, , k_{ij} = k_{ji} \, . \tag{14}$$

Based on the inequality of entropy it results:

$$-\frac{1}{T_o} k_{ij}\vartheta_{,i}\vartheta_{,j} - \omega\dot{\sigma}^2 \le 0 \quad . \tag{15}$$

The equation of the equilibrated force (12) is motivated in [16] and [17] using an argument of the variational type and the equations (10) and (13) have a shape similar to those of the classical case. In order to complete the mixed initial-boundary value problem from the context of the theory of micropolar thermoelastic bodies with voids, we must introduce the the boundary relations and initial data. Regarding the boundary conditions, we must indicate the supplementary data for the specific surfaces included in the boundary ∂D of the body D and, also, for an interval of time on which is defined the solution. Further, we must give the initial value of the temperature. As such, we impose the next initial data:

$$u_m(x,0)=u_m^0(x)\,,\dot{u}_m(x,0)=u_m^1(x)\,,x\in\overline{B}\,, \tag{16}$$

$$\varphi_m(x,0)=\varphi_m^0(x)\,,\dot{\varphi}_m(x,0)=\varphi_m^1(x)\,,x\in\overline{B}\,, \tag{17}$$

$$\vartheta(x,0)=\vartheta^0(x)\,,\sigma(x,0)=\sigma^0(x)\,,\dot{\sigma}(x,0)=\sigma^1(x)\,,x\in\overline{B}\,, \tag{18}$$

and the boundary relations:

$$u_m=\overline{u}_m \text{ on } \partial D_1\times[0,t_o),\ t_k\equiv\tau_{kl}n_l=\bar{t}_m \text{ on } \partial D_1^c\times[0,t_o); \tag{19}$$

$$\varphi_m=\overline{\varphi}_m \text{ on } \partial D_2\times[0,t_o),\ m_k\equiv\mu_{kl}n_l=\overline{m}_k \text{ on } \partial D_2^c\times[0,t_o); \tag{20}$$

$$\sigma=\overline{\sigma} \text{ on } \partial D_3\times[0,t_o),\ h\equiv h_m n_m=\overline{h} \text{ on } \partial D_3^c\times[0,t_o); \tag{21}$$

$$\vartheta=\overline{\vartheta}_m \text{ on } \partial D_4\times[0,t_o),\ q\equiv q_m n_m=\overline{q} \text{ on } \partial D_4^c\times[0,t_o), \tag{22}$$

where ∂D_1, ∂D_2, ∂D_3 and ∂D_4 with respective complements $\partial D_1^c, \partial D_2^c, \partial D_3^c$ and ∂D_4^c are surfaces from ∂D, $n=(n_m)$ is the unit normal outward to to the exterior of ∂D, t_o is a fixed initial time which can be infinite, u_m^0, u_m^1, φ_m^0, φ_m^1, ϑ^0, σ^0, σ^1, $\overline{u}_m$, $\bar{t}_m$, $\overline{\varphi}_m$, $\overline{m}_m$, $\overline{\sigma}$, $\overline{\vartheta}$, $\overline{q}$ and $\overline{h}$ are given functions in all their domains of definition.

The system of fields $(u_m,\varphi_m,\sigma,\vartheta)$ is called solution for the mixed initial-boundary value problem in the context of theory of thermoelastic micropolar bodies with pores, if it satisfies the system of equations (10-13) for all $(t,x)\in\Omega_o=[0,t_o)$, the boundary relations (16-18) and the initial data (19-22). With the help of equations (3-8) and (9), from equations (10-13) we are led to next system of equations:

$$\rho\ddot{u}_{ij}=(A_{ijmn}e_{mn}+B_{ijmn}\varepsilon_{mn}+B_{ij}\sigma+D_{ijk}\sigma_{,k}-\beta_{ij}\vartheta)_{,j}+\rho F_m \quad ; \tag{23}$$

$$\begin{aligned}I_{ij}\ddot{\varphi}_j&=(B_{mnij}e_{mn}+C_{ijmn}\varepsilon_{mn}+C_{ij}\sigma+E_{ijk}\sigma_{,k}-\alpha_{ij}\vartheta)_{,j}+\\&+\varepsilon_{ijk}(A_{jkmn}e_{mn}+B_{jkmn}\varepsilon_{mn}+B_{jk}\sigma+D_{jkm}\sigma_{,m}-\beta_{jk}\vartheta)+\rho M_m \quad ;\end{aligned} \tag{24}$$

$$\begin{aligned}\rho\kappa\ddot{\sigma}&=(D_{mni}e_{mn}+E_{mni}\varepsilon_{mn}+d_m\sigma+A_{ij})\sigma_{,j}-a_m\vartheta)_{,j}+\\&+\rho L-B_{ij}e_{ij}-C_{ij}\varepsilon_{ij}-\xi\sigma-d_m\sigma_{,i}+m\vartheta \quad ;\end{aligned} \tag{25}$$

$$a\dot{\vartheta}=\frac{1}{\rho T_o}\left(k_{ij}\vartheta_{,j}\right)_{,i}+\frac{1}{T_o}r-\beta_{ij}\dot{e}_{ij}-\alpha_{ij}\dot{\varepsilon}_{ij}-m\dot{\sigma}-a_m\dot{\sigma}_{,i}\,. \tag{26}$$

3. BASIC RESULTS

Our first main result from this section is regarding the uniqueness of solution for the above defined mixed problem. To this aim we will use an energetic method. Assume that our problem admits two solutions $\left(u_i^{(1)},\varphi_i^{(1)},\sigma^{(1)},\vartheta^{(1)}\right)$ and $\left(u_i^{(2)},\varphi_i^{(2)},\sigma^{(2)},\vartheta^{(2)}\right)$ which correspond to the same solid *D*, to the same volume force F_i, the same volume couple M_i, the same extrinsic force *L* and the same supply of the heat *r*. Each solution has a suitable set of boundary relations and a suitable set of initial data, similar to those from of (16-22). Taking into account the linearity of our problem (the linearity of the differential equations and the linearity of conditions), we can easy deduce that the difference of two solutions satisfies also our problem, but this solution is corresponding to zero volume force, volume couple, supply of the heat, zero extrinsic body force and zero initial and boundary data.

We will denote by $\left(\overline{u}_i,\overline{\varphi}_i,\overline{\sigma},\overline{\vartheta}\right)$the difference of the above two solutions, namely:

$$\overline{u}_i = u_i^{(2)} - u_i^{(1)} \quad , \quad \overline{\varphi}_i = \overline{\varphi}_i^{(2)} - \overline{\varphi}_i^{(1)} \quad , \quad \overline{\sigma} = \sigma^{(2)} - \sigma^{(1)} \quad , \quad \overline{\vartheta} = \vartheta^{(2)} - \vartheta^{(1)} \quad . \tag{27}$$

All quantities corresponding to the difference of the two solutions will also be marked with a superposed bar, for instance, $\overline{\tau}_{ij} = \tau_{ij}^{(2)} - \tau_{ij}^{(1)}$.

As such, the system of differential equations for the difference solutions becomes:

$$\overline{\tau}_{ij,j} = \rho\ddot{\overline{u}}_m \quad , \tag{28}$$

$$\overline{\mu}_{ij,j} + \varepsilon_{ijk}\overline{\tau}_{jk} = I_{ij}\ddot{\overline{\varphi}}_j \quad , \tag{29}$$

$$\overline{h}_{i,i} + \overline{g} = \rho\kappa\ddot{\overline{\sigma}} \quad , \tag{30}$$

$$\rho T_0\dot{\overline{\eta}} = \overline{q}_{i,i} \quad . \tag{31}$$

Now, we will formulate and demonstrate the result of uniqueness. To this aim we introduce the Biot's potential

$$U = \rho\left(\varepsilon - T_0\eta\right) \quad , \tag{32}$$

where η is the entropy and ε is the density of internal energy.
Taking into account the Helmholtz's function, that is, the density of energy function Ψ, we deduce that:

$$\psi = \varepsilon - T\eta \, . \tag{33}$$

Now, by considering (32) and (33) we can eliminate ε and obtain:

$$\begin{gathered} U = \frac{1}{2}A_{ijmn}e_{ij}e_{mn} + B_{ijmn}e_{ij}\varepsilon_{mn} + \frac{1}{2}C_{ijmn}\varepsilon_{ij}\varepsilon_{mn} + B_{ij}\sigma e_{ijn} + C_{ij}\sigma\varepsilon_{ijn} + D_{ijk}\sigma_{,k}e_{ij} + E_{ijk}\sigma_{,k}\varepsilon_{ij} + \\ + d_i\sigma\sigma_{,i} - \frac{1}{2}a\vartheta^2 + \frac{1}{2}\xi\sigma^2 + \frac{1}{2}A_{ij}\sigma_{,i}\sigma_{,j} - \frac{1}{2}\omega\dot{\sigma}^2 \quad . \end{gathered} \tag{34}$$

Let us define the kinetic energy *K* by:

$$K = \frac{1}{2}\left(\rho\dot{u}_i\,\dot{u}_i + I_{ij}\dot{\varphi}_i\dot{\varphi}_j + k\dot{\sigma}^2\right) \quad . \tag{35}$$

The estimation from the following theorem is an auxiliary result necessary to demonstrate the theorem of uniqueness.

Theorem 1. *Let us an arbitrary solution* $(u_m, \varphi_m, \sigma, \vartheta)$ *of our mixed problem, defined by the system of equations (23-26) and the conditions (16-22).*
Then we have the following form of the equation of energy:

$$\frac{d}{dt}\int_B (U+K)dV = \int_B \left(\rho F_i \dot{u}_i + I_{ij} M_i \dot{\varphi}_j + \frac{\rho}{T_0} r\vartheta - \frac{1}{T_0} q_i \vartheta_i + f\dot{\sigma} \right) dV +$$
$$+ \int_{\partial D} \left(\tau_{ij}\dot{u}_j + \mu_{ij}\dot{\varphi}_j + h_i\dot{\sigma} + \frac{1}{T_0} q_i \vartheta \right) n_i dA \quad . \tag{36}$$

Proof. Considerind the constitutive relations (3-8) and taking into account the relations of symmetry (14), we are led to following identity:

$$\tau_{ij}\dot{e}_{ij} + \mu_{ij}\dot{\varepsilon}_{ij} + h_i\dot{\sigma}_{,i} - g\dot{\sigma} - \frac{\rho}{T_0}\eta\dot{\vartheta} = \frac{1}{2}\frac{\partial}{\partial t}(A_{ijmn} e_{ij} e_{mn} + 2B_{ijmn} e_{ij}\varepsilon_{mn} + C_{ijmn}\varepsilon_{ij}\varepsilon_{mn} +$$
$$+ 2B_{ij}\sigma e_{ij} + 2C_{ij}\sigma\varepsilon_{ij} + D_{ijk}\sigma_{,k} e_{ij} + E_{ijk}\sigma_{,k}\varepsilon_{ij} - 2\beta_{ij}\vartheta e_{ij} - 2\alpha_{ij}\vartheta\varepsilon_{ij} - m\vartheta\sigma + \tag{37}$$
$$+ 2d_i\sigma\sigma_{,i} + 2a_i\vartheta\sigma_{,i} - a\vartheta^2 + \xi\sigma^2 + \frac{1}{2}A_{ij}\sigma_{,i}\sigma_{,j}) + \omega\dot{\sigma}^2 \quad .$$

Now, we will use the motion equations (10), the energy equation (13), the equation of the equilibrated forces (12) and the kinematic relations (9) in order to obtain the following equality:

$$\tau_{ij}\dot{e}_{ij} + \mu_{ij}\dot{\varepsilon}_{ij} + h_i\dot{\sigma}_{,i} - g\dot{\sigma} - \frac{\rho}{T_0}\eta\dot{\vartheta} =$$
$$= \left(\tau_{ij}\dot{u}_j + \mu_{ij}\dot{\varphi}_j + h_m\dot{\sigma} + \frac{q_i\vartheta}{T_0} \right)_{,i} + \rho F_m \dot{u}_i + \rho M_m \dot{\varphi}_i + \rho L\dot{\sigma} + \frac{\rho}{T_0} r\vartheta - \frac{q_i\vartheta_{,i}}{T_0} - \tag{38}$$
$$- \frac{1}{2}\frac{\partial}{\partial t}\left(\rho\dot{u}_i\dot{u}_i + I_{ij}\dot{\varphi}_i\dot{\varphi}_j + \rho\kappa\dot{\sigma}^2 \right) - \frac{\partial}{\partial t}\left(\beta_{ij} e_{ij}\vartheta + \alpha_{ij}\varepsilon_{ij}\vartheta + m\sigma\vartheta + a_m\sigma_{,i}\vartheta \right) \quad .$$

From (37) and (38) it is no dificult to obtain:

$$\frac{1}{2}\frac{\partial}{\partial t}(A_{ijmn} e_{ij} e_{mn} + 2B_{ijmn} e_{ij}\varepsilon_{mn} + C_{ijmn}\varepsilon_{ij}\varepsilon_{mn} + 2B_{ij}\sigma e_{ij} + 2C_{ij}\sigma\varepsilon_{ij} + 2D_{ijk}\sigma_{,k} e_{ij} +$$
$$2E_{ijk}\sigma_{,k}\varepsilon_{ij} - 2\beta_{ij}\vartheta e_{ij} - 2\alpha_{ij}\vartheta\varepsilon_{ij} - m\vartheta\sigma + 2d_i\sigma\sigma_{,i} + 2a_i\vartheta\sigma_{,i} - a\vartheta^2 + \xi\sigma^2 + \frac{1}{2}A_{ij}\sigma_{,i}\sigma_{,j}) +$$
$$+ \frac{1}{2}\frac{\partial}{\partial t}\left(\rho\dot{u}_i\dot{u}_i + I_{ij}\dot{\varphi}_i\dot{\varphi}_j + \rho\kappa\dot{\sigma}^2 \right) = \tag{39}$$
$$= \rho F_m \dot{u}_i + \rho M_m \dot{\varphi}_i + \rho L\dot{\sigma} + \frac{\rho}{T_0} r\vartheta - \frac{q_i\vartheta_{,i}}{T_0} + \omega\dot{\sigma}^2 + \left(\tau_{ij}\dot{u}_j + \mu_{ij}\dot{\varphi}_j + h_m\dot{\sigma} + \frac{q_i\vartheta}{T_0} \right)_{,i} \quad .$$

Finally, we integrate over *D* the equality (39) and so we obtain the estimate result (36) and we end the proof of Theorem 1. ■

The auxiliary result of Theorem 1 is the basis for proving the uniqueness result of the following theorem.

Theorem 2. *We suppose that the Biot's potential U is a non-negative function. Then the mixed problem, consistings in the equations (23-26), the boundary relations (19-22) and the initial data (16-18), admits at most one solution.*

Proof. Consider two solutions $\left(u_i^{(1)},\varphi_i^{(1)},\sigma^{(1)},\vartheta^{(1)}\right)$ and $\left(u_i^{(2)},\varphi_i^{(2)},\sigma^{(2)},\vartheta^{(2)}\right)$ of our mixed problem and suppose that these solutions correspond to the same charges F_i, M_i, L and r satisfy same boundary relations similar to those from (19-22). For the difference of these two solutions, denoted by $\left(\overline{u}_i,\overline{\varphi}_i,\overline{\sigma},\overline{\vartheta}\right)$ the estimate (36) reads:

$$\frac{d}{dt}\int_B \left(\overline{U}+\overline{K}\right)dV=\int_B\left(-\frac{1}{T_0}\overline{q}\,\overline{\vartheta}_{,i}-\omega\dot{\overline{\sigma}}^2\right)dV \quad . \tag{40}$$

If we take into account (15), we deduce:

$$\frac{d}{dt}\int_B \left(\overline{U}+\overline{K}\right)dV \le 0 \quad , \tag{41}$$

from where, by integrating on the interval $[0,t_o]$, we are led to the inequality:

$$\frac{d}{dt}\int_B \left(\overline{U}(0)+\overline{K}(0)\right)dV \ge \frac{d}{dt}\int_B \left(\overline{U}(t)+\overline{K}(t)\right)dV \; . \tag{42}$$

But the solutions $\left(u_i^{(1)},\varphi_i^{(1)},\sigma^{(1)},\vartheta^{(1)}\right)$ and $\left(u_i^{(2)},\varphi_i^{(2)},\sigma^{(2)},\vartheta^{(2)}\right)$ satisfy the same initial data, so we can deduce that the difference $\left(\overline{u}_i,\overline{\varphi}_i,\overline{\sigma},\overline{\vartheta}\right)$ satisfies the zero initial data, i.e.
$\overline{u}_i=\overline{\varphi}_i=\overline{\tau}_{ij}=\overline{\mu}_{ij}=\overline{q}_i=0$ on $\partial D\times\,[0,t_o]$,
as such, from (42) we obtain:

$$0\ge\frac{d}{dt}\int_B \left(\overline{U}(t)+\overline{K}(t)\right)dV \; . \tag{43}$$

But from (32) and (35) we deduce that the quadratic form $\overline{U}(t)$ and $\overline{K}(t)$ are positive definite and so from (43) it results that $\overline{U}(t)$ and $\overline{K}(t)$ must be null on all domain *D*. But, in this way, the difference of solutions must be null on all domain *D*, for any times $t\in[0,t_0]$. So, the proof of Theorem 2 is complete. ■

4. CONCLUSION

Within the theory of porous thermoelastic materials, we formulate and prove an uniqueness result and analyze the decay of dilatational acceleration waves.

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